

Long-period G-ring laser observations: How far can we go?

Maria Fernanda Nader-Nieto¹, Heiner Igel¹, Ana Ferreira²,
David Al-Attar³, Joachim Wassermann¹ and Ulrich Schreiber⁴

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¹ LMU Munich, Germany

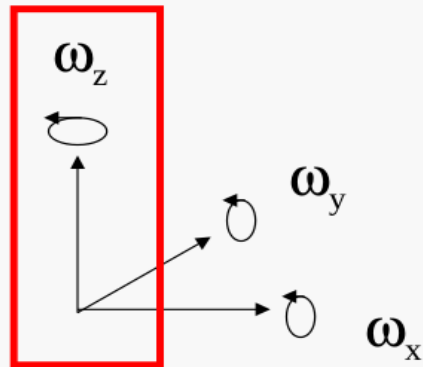
² UCL London, UK

³ Cambridge University, UK

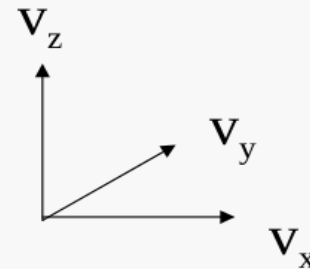
⁴ TU Munich, Germany

Rotational Motions

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \frac{1}{2} \nabla \times \underline{v} = \frac{1}{2} \begin{pmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{pmatrix}$$



Rotation rate
Rotation sensor



Ground velocity
Seismometer

Transversely polarized plane wave propagating in x-direction

$$u_y(x, t) = F(kx - \omega t) \quad c = \frac{\omega}{k}:$$

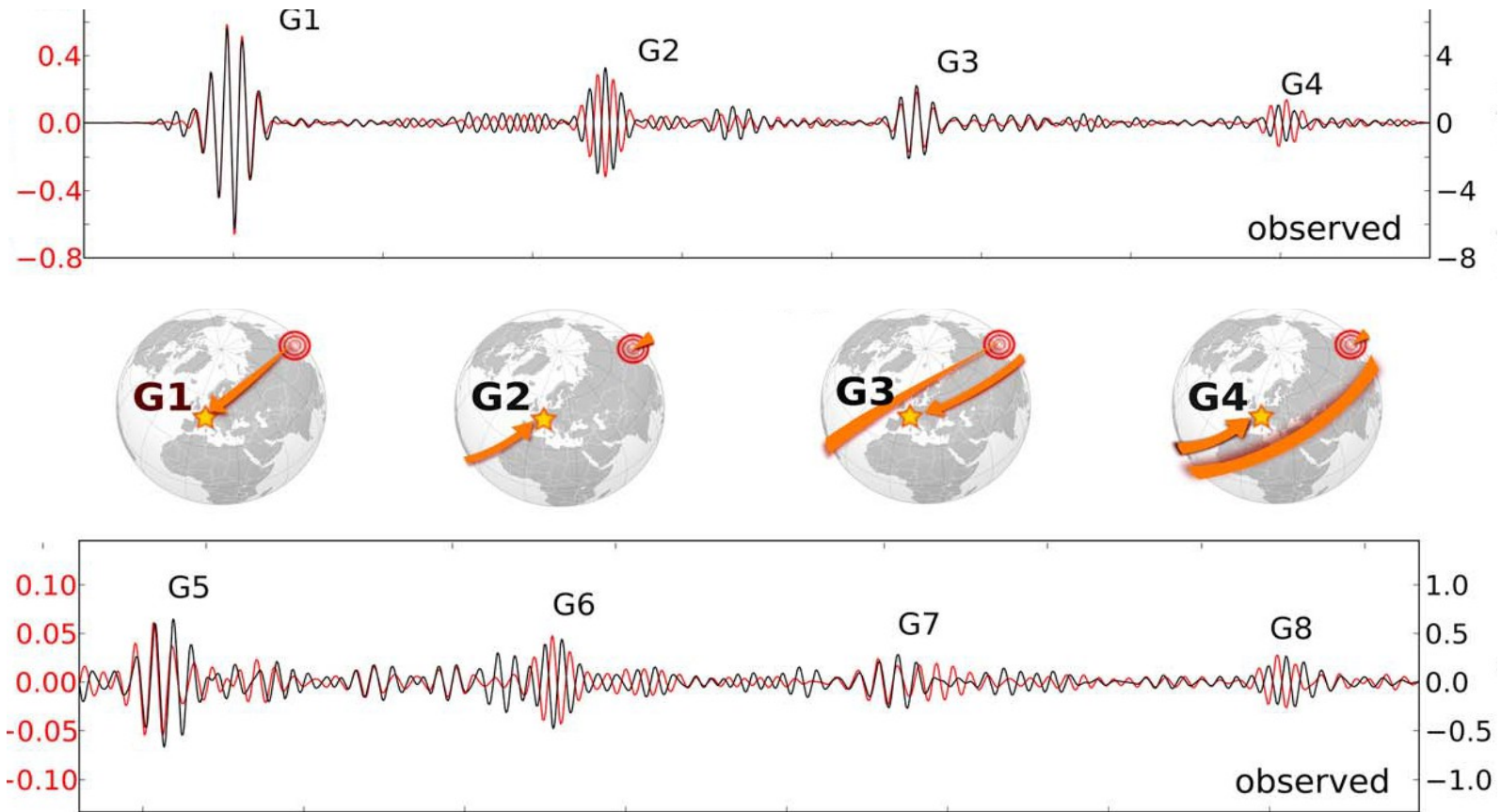
Acceleration : $a_y = \ddot{u}_y = \omega^2 F''(kx - \omega t)$

Rotation rate: $\dot{\Omega} = \frac{1}{2} \nabla \times \dot{u}_y = -\frac{1}{2} k \omega F''$

$$a(x, t) / \dot{\Omega}(x, t) = -2c$$

Rotation rate and acceleration are **in phase** and
the amplitudes scaled by two times the horizontal phase velocity.

Love waves GX traveling around the Earth



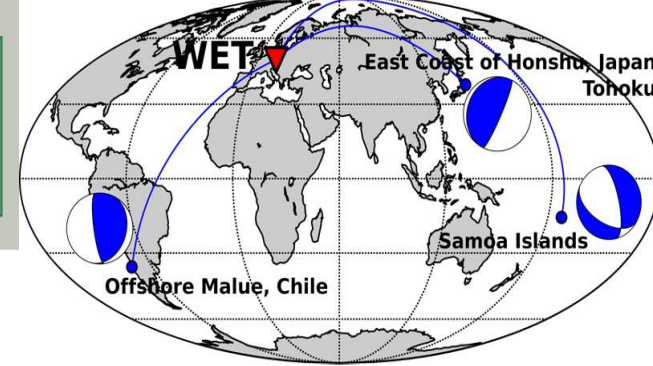
Black – seismometer

Red – ring laser

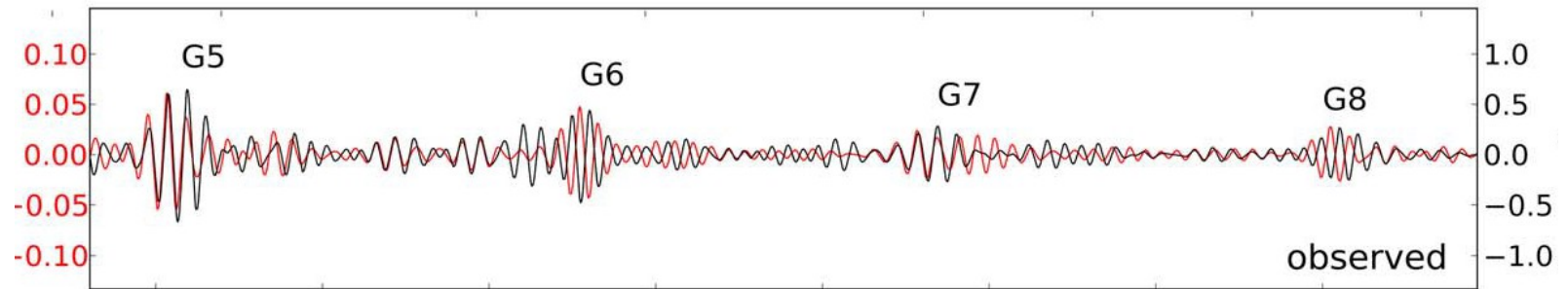
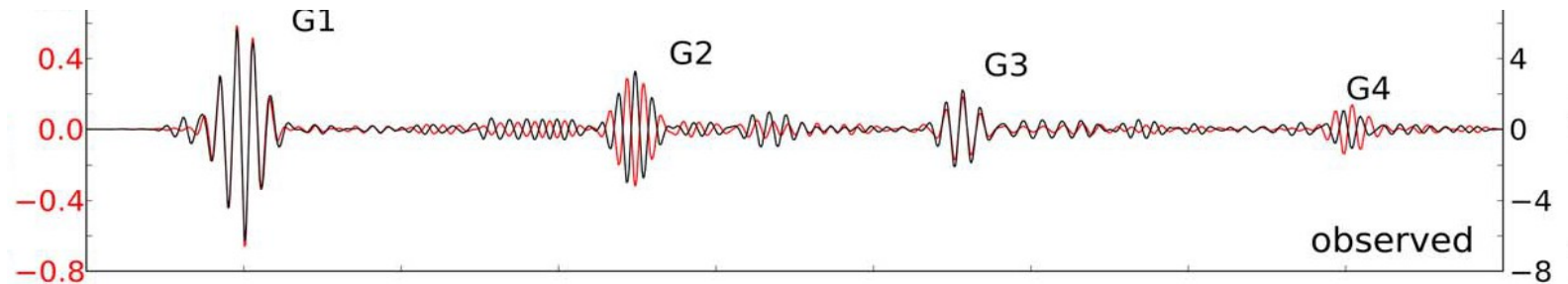
Igel et al., GRL, 2011

Nader et al. JOSE, 2012 # 5

Tohoku-Oki M9 Earthquake



Love waves GX traveling around the Earth

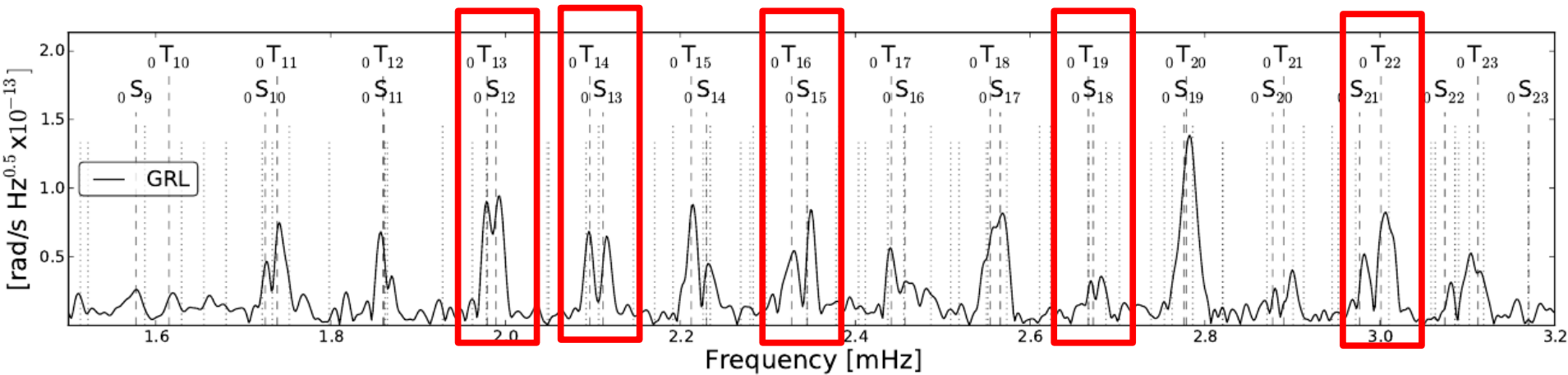


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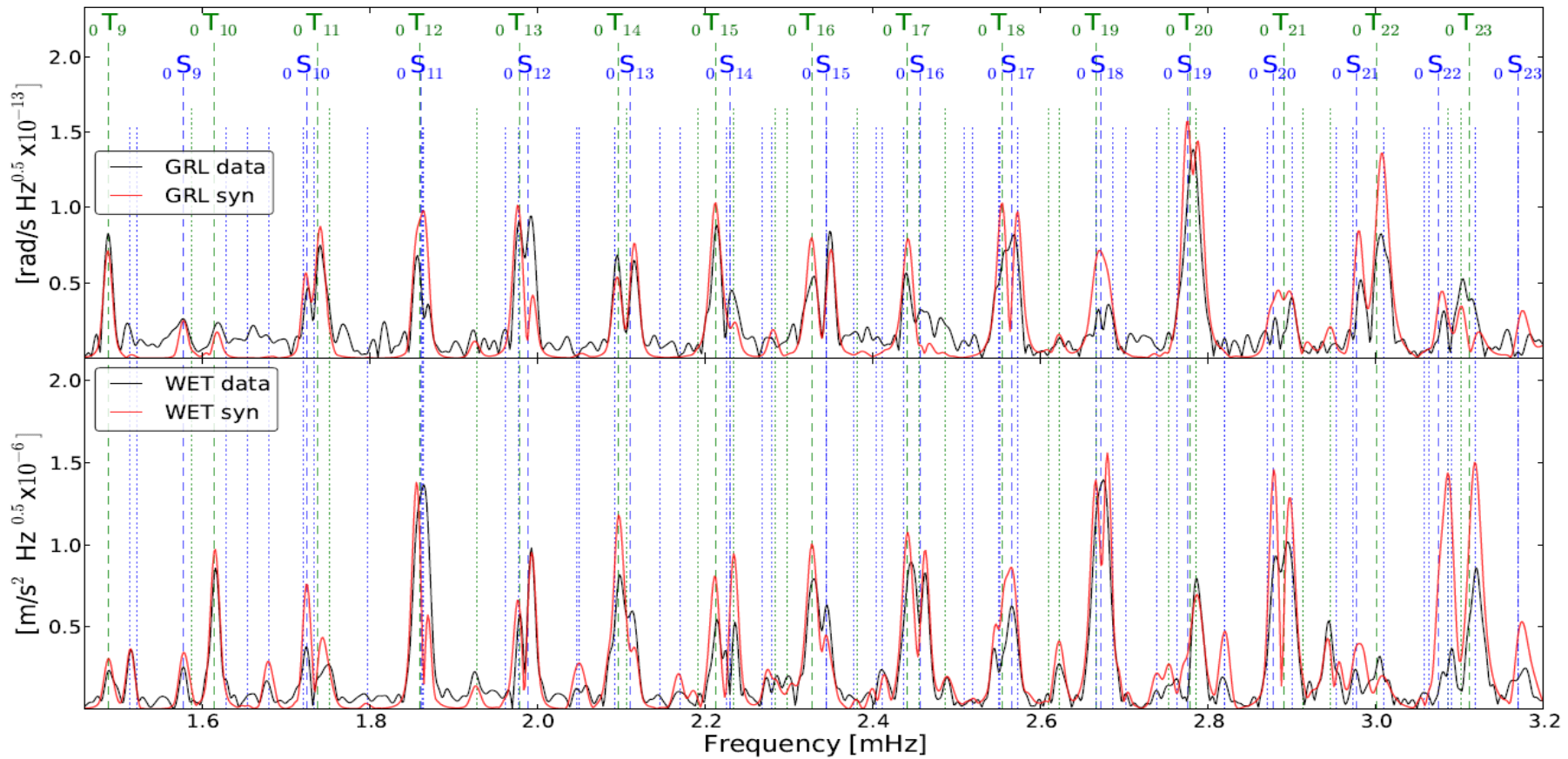
Igel et al., GRL, 2011

Nader et al. JOSE, 2012 # 6



Amplitude spectrum of 48 hour-long time series of the z-component rotations after the Mw 9.0 Tohoku-Oki earthquake (2011) recorded at Wettzell, Germany by the G-ring laser (GRL).

Dashed vertical line are the toroidal and spheroidal eigenfrequencies



The relative percentage difference of the observed-synthetic correlation between the observables is 3.2% for the frequency band target of our study.

Can we detect a signature of permanent excitation of Earth's Free oscillations in the G-Ring laser data?

The ground displacement associated with a given mode of vibration
(Dahlen & Tromp 1998):

$$S_i(x) = {}_n \mathcal{D}_\ell(r\theta\phi) Y_{lm}(\theta, \phi)$$

Where:

$$\mathcal{D} = U\hat{r} + \hat{\Theta}k^{-1}[V\partial_\Theta + W(\sin\Theta)^{-1}\partial_\Phi] + \hat{\Phi}k^{-1}[V(\sin\Theta)^{-1}\partial_\Phi - W\partial_\Theta]$$

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Rotational motions are the curl of the wave field:

$$[\nabla \times \mathcal{D}]_{\hat{r}} = \frac{\hat{r}}{r \sin \Theta} k^{-1} [\partial_\Theta^2 (-W \sin \Theta) + \partial_\Phi^2 (W \sin \Theta^{-1})]$$

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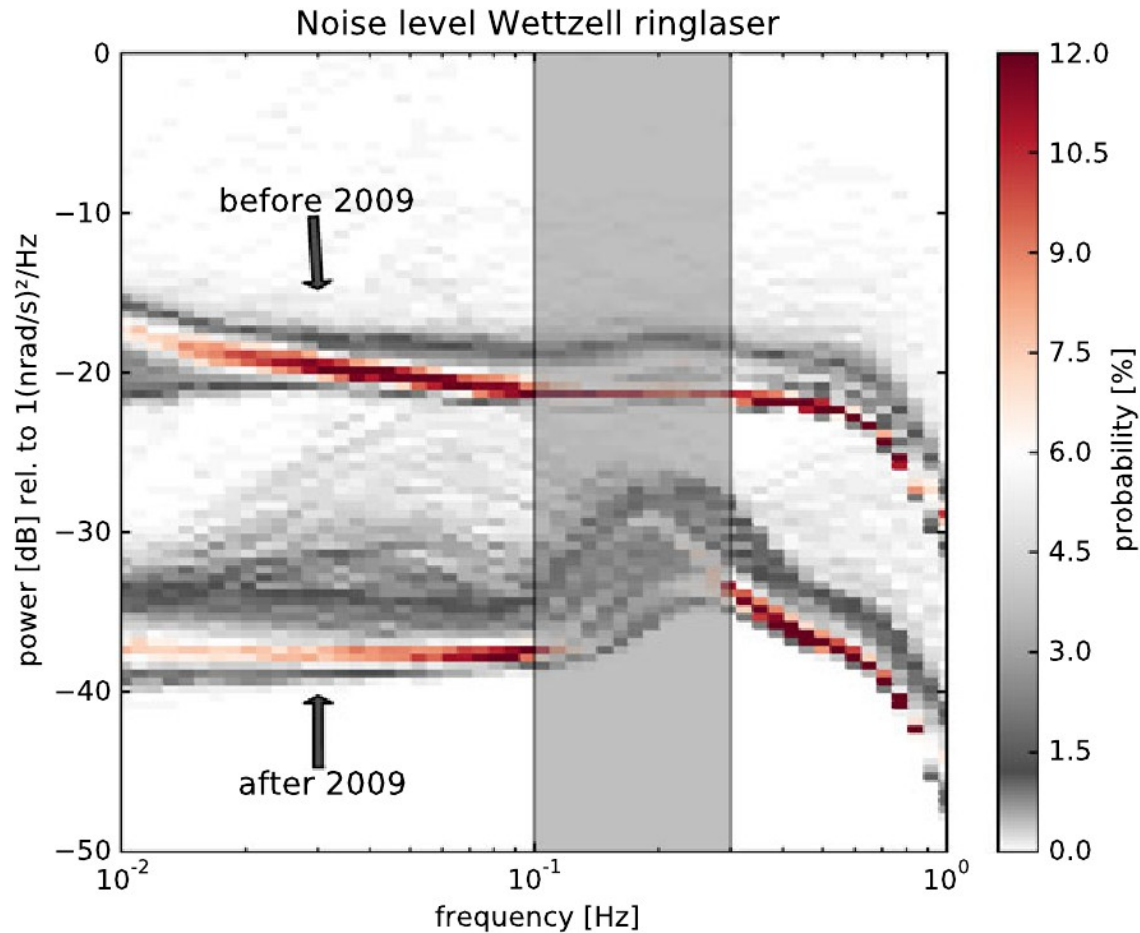
$$S_i(x) = {}_n \mathcal{D}_\ell(r\theta\phi) Y_{lm}(\theta, \phi)$$

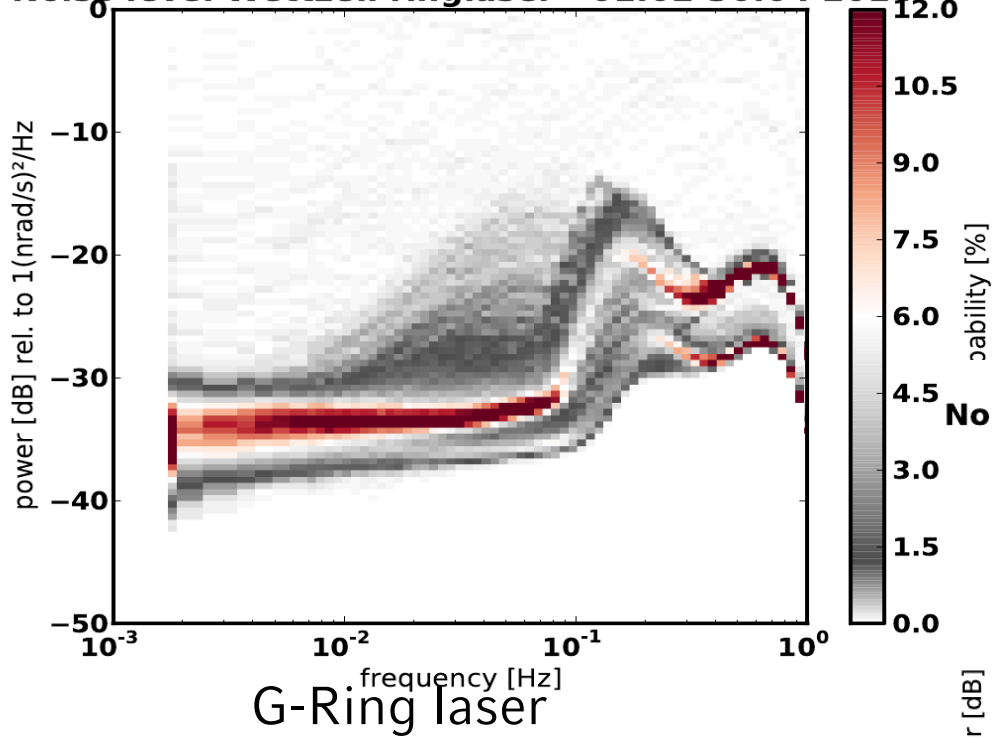
Where:

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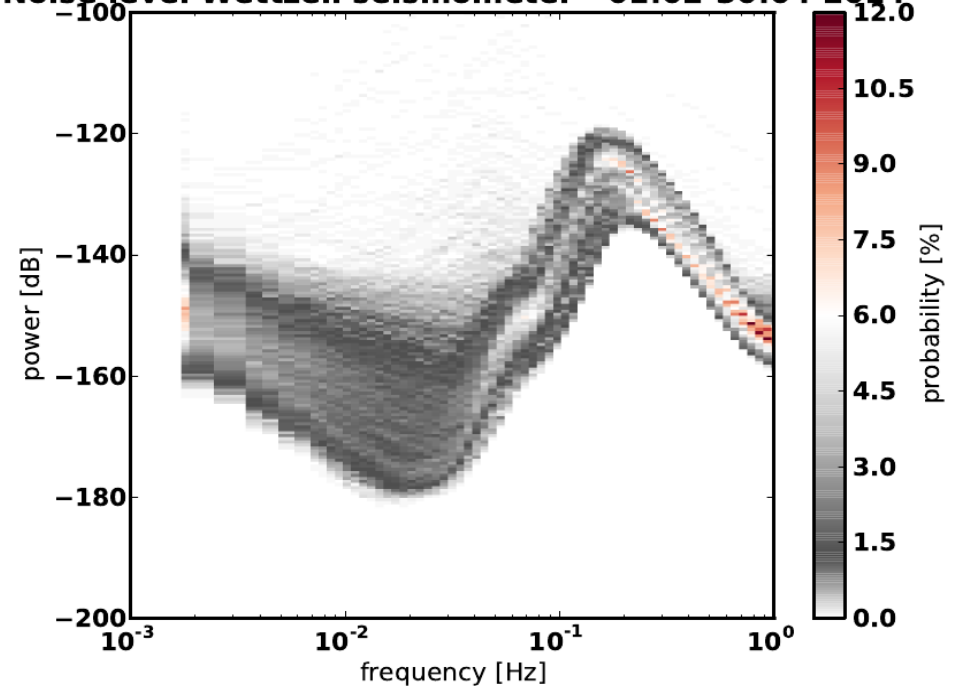
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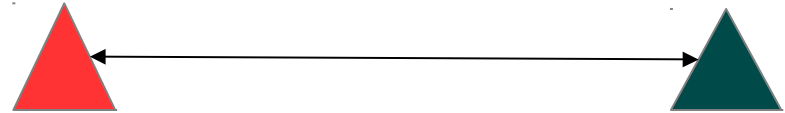
Noise level Wettzell ringlaser - 01.02-30.04 2014

STS-2 seismometer
N-S component

Noise level Wettzell seismometer - 01.02-30.04 2014

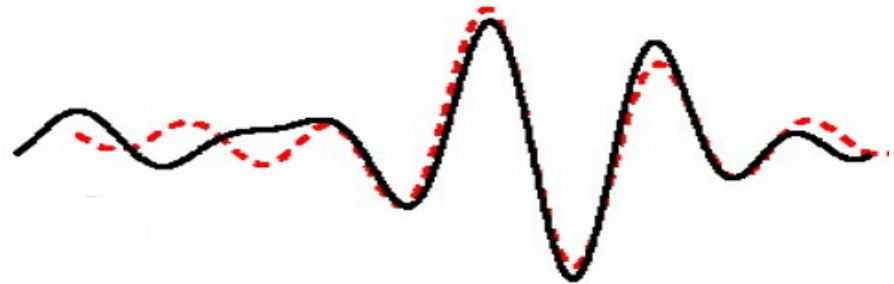
Can we observe Love waves
in the ring laser noise?

G-ring



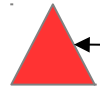
Black – Transv. acc
Red – rotational rate

Rotational rate and transverse
Acceleration are in phase.

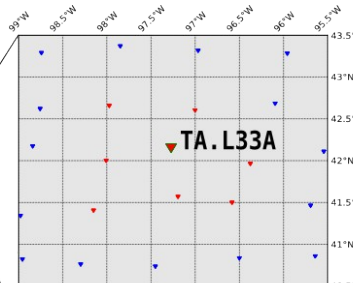
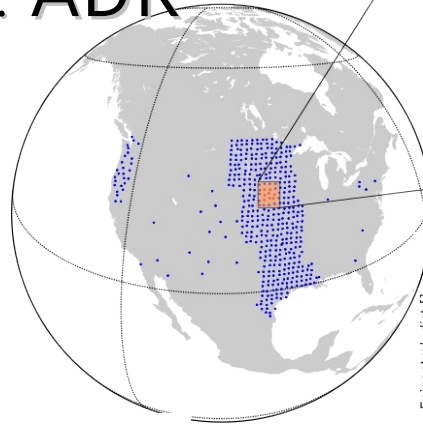


Alternative: **ADR**

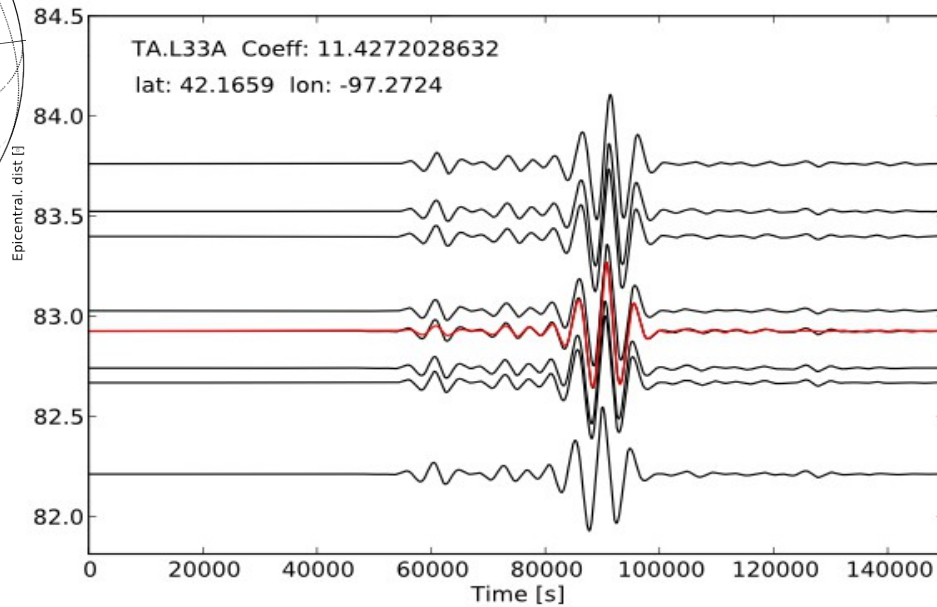
G-ring



?



Tohoku-Oki M9



Black – Transv. acc

Red – rotational rate

Method : Time domain analysis of Earth's long-period background (Ekstroem, 2001)

Standing Love waves (200-400s)

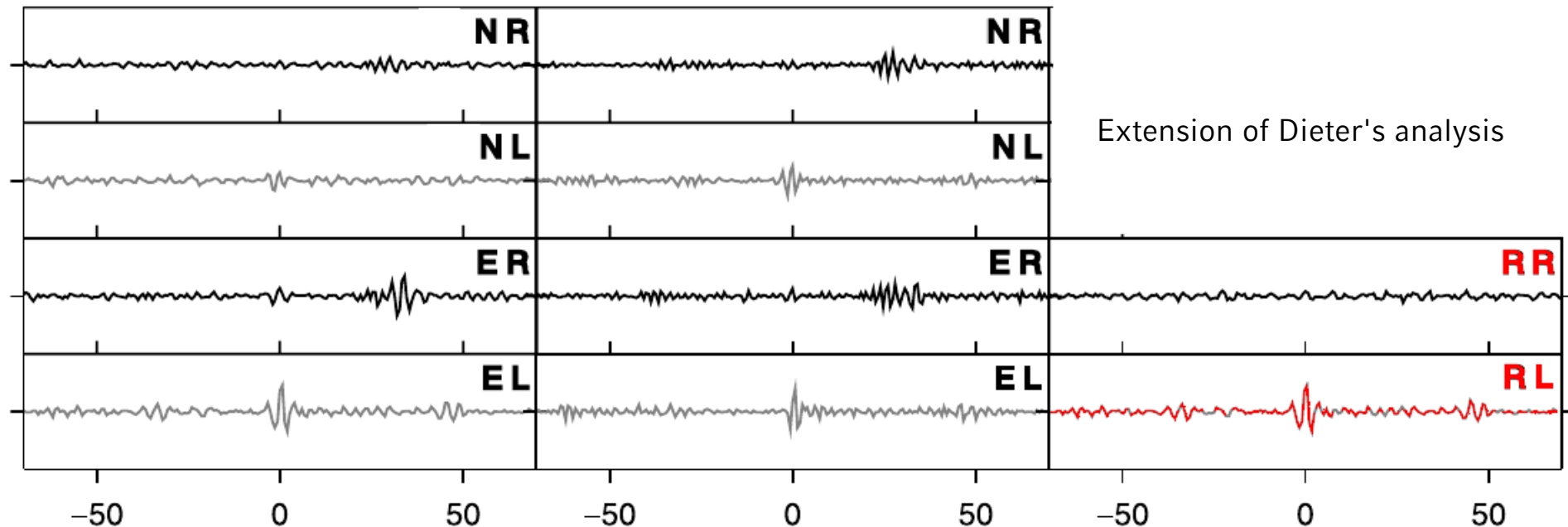
Based on autocorrelation functions and the great circle operator

Transforms $G_n \rightarrow G_{n+2}$ correlating of an original seismogram with a copy to which the operator was applied

BFO

WET STS2

G-ring



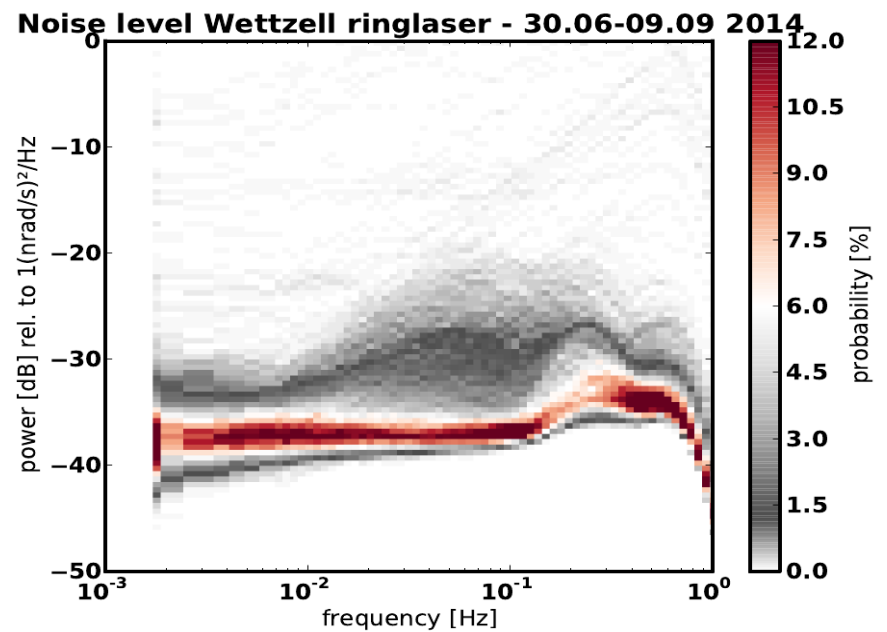
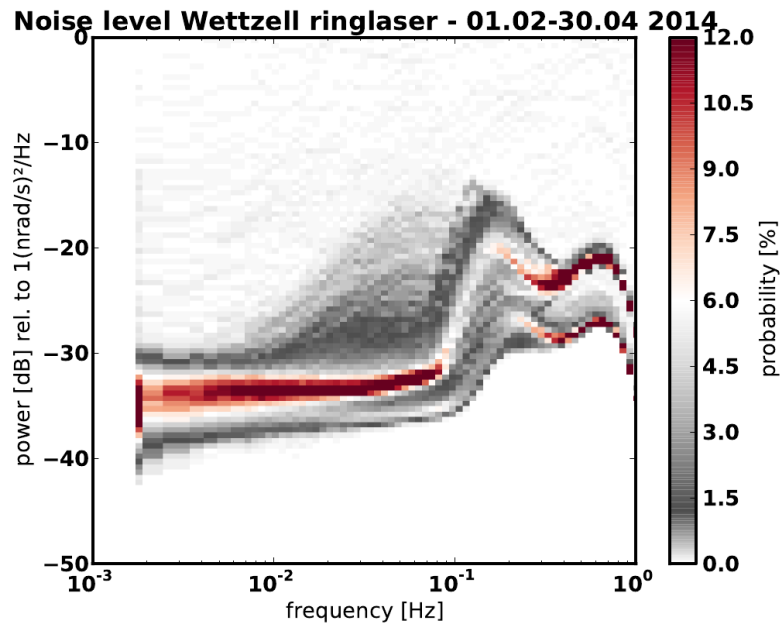
THE GOAL!

Searching for signs of continuously excited normal modes in the G-ring laser noise.

Can we detect permanent excitation of normal modes in the Ring laser data?

And if so ...

Does an answer to this question can give us more insights about the mechanism that generates the horizontal global Hum?



Strain Rotation coupling:

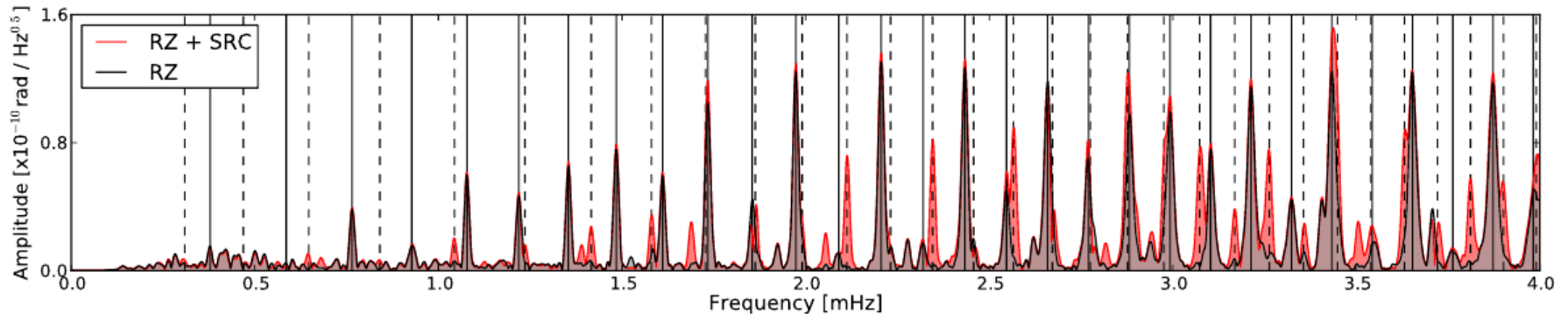
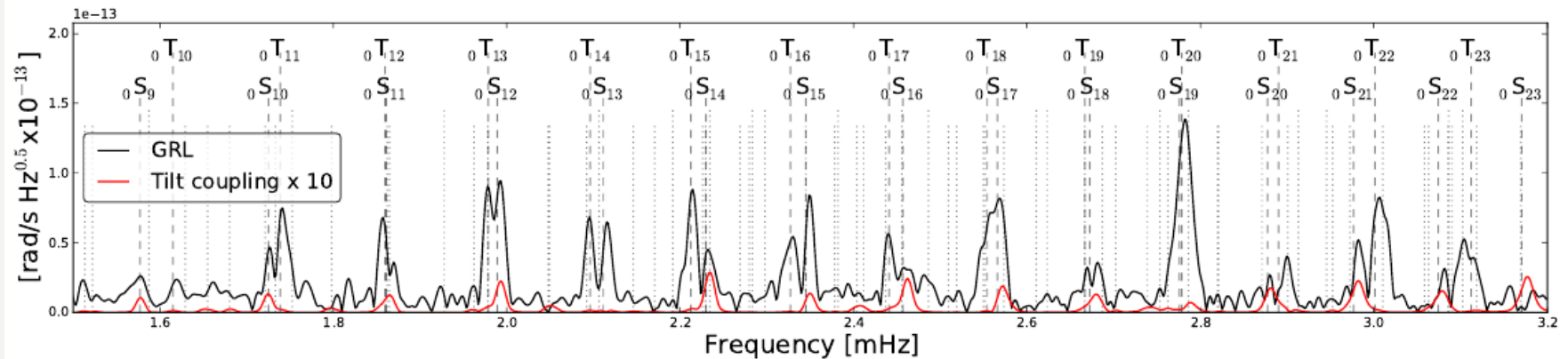


Fig. 9 Amplitude spectrum for rotation around z -axis for 36 h of synthetic data for the $M9.0$ Tohoku, Japan, earthquake recorded in Wettzell, Germany. Pure rotation with main peaks at toroidal modes (*vertical lines mark 0Ti*

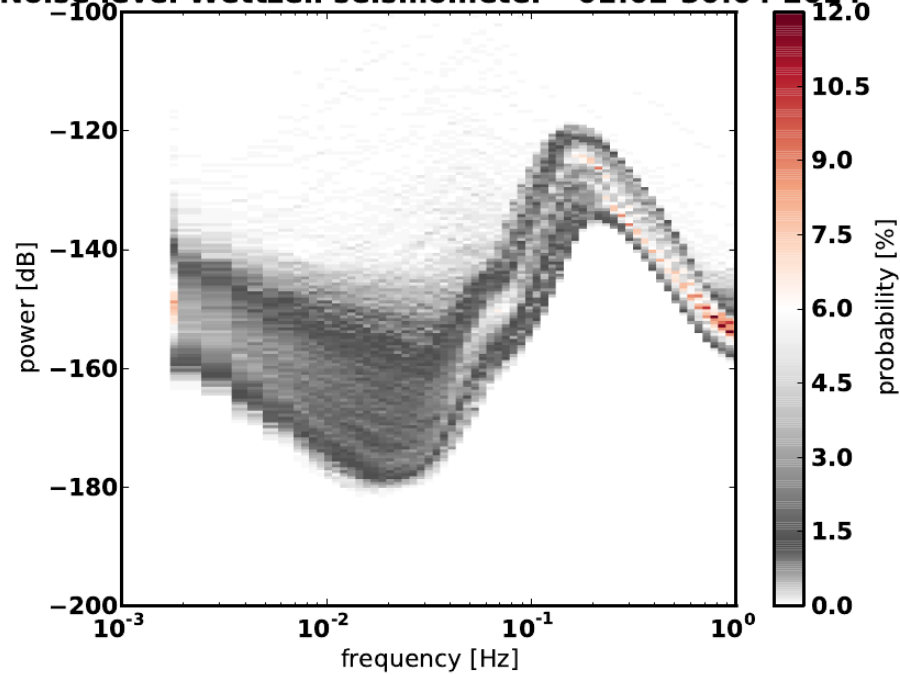
modes); strain rotation coupling (here constants are chosen $c_{NZ} = c_{EZ} = -c_{SZ} = 0.2$) can cause additional peaks at spheroidal modes (*dashed lines mark 0Si modes*)

Tilt-Ring Laser Coupling

$$\dot{\Omega}_{tilt} = -\dot{\Omega}_P \Omega_E \cos \left(\Lambda - \frac{\Omega_E}{2} \right)$$



Noise level Wettzell seismometer - 01.02-30.04 2014



Noise level Wettzell seismometer - 30.06-09.09 2014

